
Contents

Preface **v**

PART 1 RANDOM SIGNALS BACKGROUND **1**

1 Probability and Random Variables: A Review **3**

- 1.1 Random Signals 3
- 1.2 Intuitive Notion of Probability 4
- 1.3 Axiomatic Probability 5
- 1.4 Random Variables 8
- 1.5 Joint and Conditional Probability, Bayes Rule and Independence 9
- 1.6 Continuous Random Variables and Probability Density Function 13
- 1.7 Expectation, Averages, and Characteristic Function 15
- 1.8 Normal or Gaussian Random Variables 18
- 1.9 Impulsive Probability Density Functions 22
- 1.10 Joint Continuous Random Variables 23
- 1.11 Correlation, Covariance, and Orthogonality 26
- 1.12 Sum of Independent Random Variables and Tendency Toward Normal Distribution 28
- 1.13 Transformation of Random Variables 32
- 1.14 Multivariate Normal Density Function 37
- 1.15 Linear Transformation and General Properties of Normal Random Variables 40
- 1.16 Limits, Convergence, and Unbiased Estimators 43
- 1.17 A Note on Statistical Estimators 46

2 Mathematical Description of Random Signals 57

2.1	Concept of a Random Process	57
2.2	Probabilistic Description of a Random Process	60
2.3	Gaussian Random Process	62
2.4	Stationarity, Ergodicity, and Classification of Processes	63
2.5	Autocorrelation Function	65
2.6	Crosscorrelation Function	68
2.7	Power Spectral Density Function	70
2.8	White Noise	75
2.9	Gauss–Markov Processes	77
2.10	Narrowband Gaussian Process	81
2.11	Wiener or Brownian-Motion Process	83
2.12	Pseudorandom Signals	86
2.13	Determination of Autocorrelation and Spectral Density Functions from Experimental Data	90
2.14	Sampling Theorem	95

3 Linear Systems Response, State-Space Modeling, and Monte Carlo Simulation 105

3.1	Introduction: The Analysis Problem	105
3.2	Stationary (Steady-State) Analysis	106
3.3	Integral Tables for Computing Mean-Square Value	109
3.4	Pure White Noise and Bandlimited Systems	110
3.5	Noise Equivalent Bandwidth	111
3.6	Shaping Filter	113
3.7	Nonstationary (Transient) Analysis	114
3.8	Note on Units and Unity White Noise	118
3.9	Vector Description of Random Processes	121
3.10	Monte Carlo Simulation of Discrete-Time Processes	128
3.11	Summary	130

PART 2 KALMAN FILTERING AND APPLICATIONS 139

4 Discrete Kalman Filter Basics 141

4.1	A Simple Recursive Example	141
4.2	The Discrete Kalman Filter	143

4.3	Simple Kalman Filter Examples and Augmenting the State Vector	148
4.4	Marine Navigation Application with Multiple-Inputs/Multiple-Outputs	151
4.5	Gaussian Monte Carlo Examples	154
4.6	Prediction	159
4.7	The Conditional Density Viewpoint	162
4.8	Re-cap and Special Note On Updating the Error Covariance Matrix	165

5 Intermediate Topics on Kalman Filtering **173**

5.1	Alternative Form of the Discrete Kalman Filter – the Information Filter	173
5.2	Processing the Measurements One at a Time	176
5.3	Orthogonality Principle	178
5.4	Divergence Problems	181
5.5	Suboptimal Error Analysis	184
5.6	Reduced-Order Suboptimality	188
5.7	Square-Root Filtering and U-D Factorization	193
5.8	Kalman Filter Stability	197
5.9	Relationship to Deterministic Least Squares Estimation	198
5.10	Deterministic Inputs	201

6 Smoothing and Further Intermediate Topics **207**

6.1	Classification of smoothing Problems	207
6.2	Discrete Fixed-Interval Smoothing	208
6.3	Discrete Fixed-Point Smoothing	212
6.4	Discrete Fixed-Lag Smoothing	213
6.5	Adaptive Kalman Filter (Multiple Model Adaptive Estimator)	216
6.6	Correlated Process and Measurement Noise for the Discrete Filter—Delayed-State Filter Algorithm	226
6.7	Decentralized Kalman Filtering	231
6.8	Difficulty with Hard-Bandlimited Processes	234
6.9	The Recursive Bayesian Filter	237

7 Linearization, Nonlinear Filtering, and Sampling Bayesian Filters **249**

7.1	Linearization	249
7.2	The Extended Kalman Filter	257
7.3	“Beyond the Kalman Filter”	260

7.4	The Ensemble Kalman Filter	262
7.5	The Unscented Kalman Filter	265
7.6	The Particle Filter	269

8 The “Go-Free” Concept, Complementary Filter, and Aided Inertial Examples 284

8.1	Introduction: Why Go Free of Anything?	284
8.2	Simple GPS Clock Bias Model	285
8.3	Euler/Goad Experiment	287
8.4	Reprise: GPS Clock-Bias Model Revisited	289
8.5	The Complementary Filter	290
8.6	Simple Complementary Filter: Intuitive Method	292
8.7	Kalman Filter Approach—Error Model	294
8.8	Kalman Filter Approach—Total Model	296
8.9	Go-Free Monte Carlo Simulation	298
8.10	INS Error Models	303
8.11	Aiding with Positioning Measurements—INS/DME Measurement Model	307
8.12	Other Integration Considerations and Concluding Remarks	309

9 Kalman Filter Applications to the GPS and Other Navigation Systems 318

9.1	Position Determination with GPS	318
9.2	The Observables	321
9.3	Basic Position and Time Process Models	324
9.4	Modeling of Different Carrier Phase Measurements and Ranging Errors	330
9.5	GPS-Aided Inertial Error Models	339
9.6	Communication Link Ranging and Timing	345
9.7	Simultaneous Localization and Mapping (SLAM)	348
9.8	Closing Remarks	352

APPENDIX A Laplace and Fourier Transforms 365

APPENDIX B The Continuous Kalman Filter 371

Index 379